Mathematics (Subsidiary)
Part I.

Dated - 17/04/21 (Equivalence Relation)

Equivalence relation: — A relation R difined on a set A is an equivalence ralation iff it satisfies all the following three conditions.

(i) R is reflexive
ie. $aRa \; \forall a \in A$

(ii) R is symmetric
ie. $aRb \Rightarrow bRa \; \forall a, b \in A$

(iii) R is transitive
ie. aRb and $bRc \Rightarrow aRc \; \forall a, b \in A$.

Theorem: — Show that inverse of ~~the~~ an equivalence relation is also an equivalence relation:

Proof: — Given that R is an equivalence relation then in set X then R must be reflexive, symmetric and transitive
let $a, b, c \in X$ then we have

for R^{-1}

(i) Reflexive: — $(a, a) \in R^{-1}$, for $(a, a) \in R \; \forall a \in X$
$\Rightarrow (a, a) \in R^{-1}$
R^{-1} is reflexive.

(ii) Symmetric :- $(a,b) \in R^{-1} \Rightarrow (b,a) \in R^{-1}$

for $(a,b) \in R^{-1} \Rightarrow (b,a) \in R$

$\Rightarrow (a,b) \in R$ ($\because R$ is symmetric)

$\Rightarrow (b,a) \in R^{-1}$

$\therefore R^{-1}$ is symmetric.

also $(a,b) \in R^{-1} \Rightarrow (a,b) \in R \therefore R^{-1} = R$ in this case.

(iii) Transitive :-

$(a,b), (b,a) \in R^{-1}$

$\Rightarrow (a,c) \in R^{-1}$

We have $(a,b), (b,c) \in R^{-1}$

$\Rightarrow (b,a)\ (c,b) \in R$

$\Rightarrow (c,b)\ (b,a) \in R$

$\Rightarrow (c,a) \in R \Rightarrow (a,c) \in R^{-1}$

$\therefore R^{-1}$ is transitive

$\therefore R^{-1}$ is an equivalence relation in X.

— Proved.